FAST NONLINEAR WAVEFORM ESTIMATION FOR LARGE DISTRIBUTED NETWORKS

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Abstract

A novel nonlinear waveform estimation method based on the Asymptotic Waveform Evaluation is presented. The method generates transient waveform estimation of large, distributed networks with arbitrary terminations, while using neither convolution nor numerical inversion techniques, both of which are found to be expensive for other methods. These are avoided due to the exponential form of the fundamental modes of the linear system. The linear network analysis speedup of waveform estimation, already demonstrated, is here supplemented for the nonlinear case.

1.Introduction

Transient analysis of large networks containing both lumped and distributed elements including lossy, coupled transmission lines with arbitrary nonlinear terminations has proven to be CPU intensive. Several methods have been proposed for the analysis of networks containing coupled and/or lossy transmission lines [1]-[8]. Some of these methods have been modified to apply to networks with nonlinear terminations [9]-[12]. Additionally, other new approaches have been proposed [13]-[14] for the analysis of nonlinear distributed networks. Of these methods, all except lumped-element model reductions, require time-domain convolution and/or numerical inversion from the frequency domain to the time domain.

Any convolution method used implies that for each time point calculation all previous time points must be taken into account. The CPU expense can quickly become large with increasing number of time points. This is especially true for mismatched transmission lines. Djordjevic et al. [10] have noted that convolution is the most time-consuming portion of the calculations and attempt to minimize it by quasi-matching the transmission lines. Schutt-Aine and Mittra [9] also make note of the expense of convolution which they are able to avoid, with considerable speed-up, only for the lossless case.

Additionally, most methods require some form of numerical inversion algorithms such as FFT [10] or Numerical Inversion of Laplace Transform (NILT) [7], sometimes applied in numerous iterations [14], to transform frequency points into time-domain descriptions. This is an additionally large expense for large system and with many frequency points.

In [11] a new approach was proposed for nonlinear waveform estimation which does not require numerical inversion methods. However, the method still requires convolution.

In this summary we outline a novel extension of waveform estimation which requires neither convolution nor numerical inversion methods. It is ideally suited for very large systems with many components. The method relies directly on the approximate poles and residues obtained from the Asymptotic Waveform Evaluation (AWE) [15] and its generalization (GAWE) [8] which have previously been successfully applied to linear waveform estimation. The AWE and GAWE have been shown to give an estimated transient response with 2-3 orders of magnitude speed-up over SPICE. With elimination of convolution, the nonlinear solution speed-up is increased further by 25 to 50 times. In addition, the proposed method can handle the general case where the network contains lossy, multiconductor transmission lines, whereas conventional circuit simulators such as SPICE cannot. While being fast, the method is ideal for implementation in terms of parallel computation which would increase its efficiency.

2. Response of the Linear Subnetwork

Consider a nonlinear network π which contains an arbitrary linear subnetwork. The linear subnetwork may contain distributed components. The actual frequency response of the linear subnetwork due to an impulse applied at a single input and measured at single output can be given by,

$$H(s) = \sum_{n=1}^{N} \frac{k_n}{s-p_n} \tag{1}$$

where p_n is the n-th pole of the response and k_n its corresponding residue. Distributed systems imply an infinite number of transcendental poles, ($N = \infty$). However, by applying the Generalized Asymptotic Waveform Evaluation (GAWE) technique to the modified nodal admittance matrix equations of the linear subnetwork networks [11],[16], it is possible to extract an approximate q-pole transfer function of this network.

Let the approximate response due to a single input applied only at x_i and measured at output v_j be described by,

$$H^{[i,j]}(s) = \sum_{n=1}^{q} \frac{k_n^{[i,j]}}{s-p_n^{[i,j]}} \tag{2}$$

where $p_n^{[i,j]}$ is the n-th approximate dominant pole of the response and $k_n^{[i,j]}$ its corresponding n-th approximate residue.

The poles of a system are its fundamental modes. Obtaining

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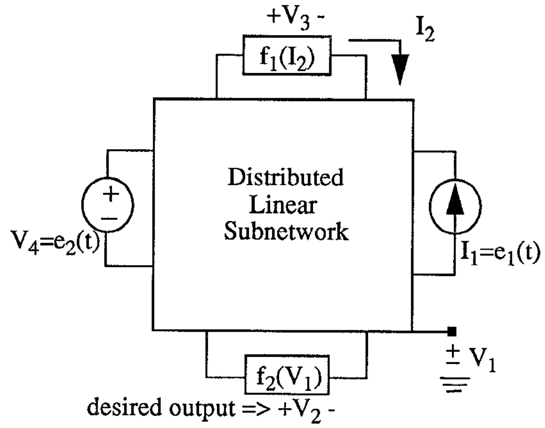
their approximation is not equivalent to simply forming a rational function approximation. It is to be noted that several improvements to the method of obtaining these fundamental modes of a linear system are currently in progress [17],[18].

We characterize the entire linear subnetwork by a minimal $M \times N$ input-output approximate transfer function description. The N inputs x_i to be described will be either voltage nodes with attached independent voltage sources or attached augmenting voltage sources in the form of nonlinear terminations where the voltage is a function of some variable(s), or branch currents with attached independent current sources or attached augmenting nonlinear terminations where the current is a function of some variable(s).

The M output ports v_j are the desired voltage nodes and branch current outputs, and all voltage nodes and branch currents which are the arguments of the nonlinear functions yielding the inputs mentioned. (See example in Figure 1).



$M \times N$ input-output characterization
N=4 inputs: V_2, V_3, V_4, I_1
M=3 outputs: V_1, V_2, I_2

Figure 1: Example Nonlinear Network.

The general $M \times N$ hybrid characterization is given by,

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_M \end{bmatrix} = H_{M \times N}(s) \begin{bmatrix} x_1 \\ x_2 \\ \\ x_N \end{bmatrix} \quad (3)$$

or,

$$\underline{V} = H_{M \times N}(s) \underline{X} \quad (4)$$

where, using (2), $H_{M \times N}(s)$ is given by,

$$\sum_{n=1}^{q} \begin{bmatrix} \frac{k_n^{[1,1]}}{s-p_n^{[1,1]}} & \frac{k_n^{[1,2]}}{s-p_n^{[1,2]}} & \cdots & \frac{k_n^{[1,M]}}{s-p_n^{[1,M]}} \\ \frac{k_n^{[2,1]}}{s-p_n^{[2,1]}} & \frac{k_n^{[2,2]}}{s-p_n^{[2,2]}} & \cdots & \frac{k_n^{[2,M]}}{s-p_n^{[2,M]}} \\ & \vdots & \vdots & \\ \frac{k_n^{[N,1]}}{s-p_n^{[N,1]}} & \frac{k_n^{[N,2]}}{s-p_n^{[N,1]}} & \cdots & \frac{k_n^{[N,M]}}{s-p_n^{[N,M]}} \end{bmatrix} \vdots \quad (5)$$

To make the mathematical analysis simpler, we assume that the same q approximate dominant poles characterize each of the input-output transfer functions; i.e. $p_n^{[i,j]} = p_n, \ \forall [x_i, v_j]$, giving,

$$H_{M \times N}(s) = \sum_{n=1}^{q} (s-p_n)^{-1} \begin{bmatrix} k_n^{[1,1]} & k_n^{[1,2]} & \cdots & k_n^{[1,M]} \\ k_n^{[2,1]} & k_n^{[2,2]} & \cdots & k_n^{[2,M]} \\ \vdots & \vdots & & \vdots \\ k_n^{[N,1]} & k_n^{[N,1]} & \cdots & k_n^{[N,M]} \end{bmatrix} \quad (6)$$

or in compact form,

$$H_{M \times N}(s) = \sum_{n=1}^{q} K_n (s-p_n)^{-1} \quad (7)$$

Without the need for numerical inversion, the approximate $M \times N$ transient impulse response is immediately given by a closed form,

$$h_{M \times N}(t) = \sum_{n=1}^{q} K_n exp(p_n t) \quad (8)$$

3. Response of the nonlinear network

The required subset response $\underline{V}(t)$ and its approximation $\hat{\underline{V}}(t)$ consists of the subset of M entries of $V(t)$ due to a general $\underline{X}(t)$ input equal to a subset of N non-zero entries of $X(t)$. $\hat{\underline{V}}(t)$ can be given by the convolution of the approximate transient $M \times N$ transfer response and the input waveforms as,

$$\hat{\underline{V}}(t) = \sum_{n=1}^{q} \int_0^t K_n exp(p_n(t-\tau)) \underline{X}(\tau) d\tau = \sum_{n=1}^{q} \hat{\underline{V}}^{(n)}(t) \quad (9)$$

In order to simplify we use two points and the trapezoidal rule for integration from $t = 0$ to $t = \Delta t$,

$$\hat{\underline{V}}^{(n)}(\Delta t) = \frac{\Delta t}{2}(K_n[\underline{X}(0) + exp(p_n \Delta t)\underline{X}(\Delta t)]) \quad (10)$$

Substituting $t = t + \Delta t$ in (9) we obtain,

$$\hat{\underline{V}}^{(n)}(t+\Delta t) = \int_0^{t+\Delta t} K_n exp(p_n(t+\Delta t-\tau)) \underline{X}(\tau) d\tau \quad (11)$$

From (9) and (11) we get,

$$\underline{\hat{V}}^{(n)}(t+\Delta t) = exp(p_n\Delta t)\left[\underline{\hat{V}}^{(n)}(t)\right] + \frac{\Delta t}{2}(K_n[\underline{X}(t+\Delta t) + exp(p_n\Delta t)\underline{X}(t)]) \tag{12}$$

The last equation is a recursive one, which says that the transient output at a time point depends only on the output at the previous time point and the present and previous input waveform time points. We have eliminated the convolution.

In the formulation of (3) we treated the nonlinear elements as augmenting sources. If we further expand $\underline{X}(t)$ in terms of its linear and nonlinear components, $\underline{e}(t)$ and $f(\underline{V}(t))$, and use $\underline{\hat{V}}(t) \approx \underline{V}(t)$, we get,

$$\underline{\hat{v}}(t+\Delta t) = C_1 + C_2 f(\underline{\hat{v}}(t+\Delta t)) \tag{13}$$

where,

$$C_1 = \sum_{n=1}^{q} exp(p_n\Delta t)\left[\underline{\hat{V}}^{(n)}(t)\right] + \frac{\Delta t}{2}\sum_{n=1}^{q} K_n\{\underline{e}(t+\Delta t) + exp(p_n\Delta t)\,[f(\underline{\hat{V}}(t)) + \underline{e}(t+\Delta t)]\} \tag{14}$$

is a constant which depends on the previous time point and the present and previous input waveform timepoints, and

$$C_2 = \frac{\Delta t}{2}\sum_{n=1}^{q} K_n \tag{15}$$

Every time point may be calculated as a solution to nonlinear equation in (13) whose constants depend only on the previous time point solution.

4. CPU considerations

The linear waveform estimation of the GAWE was shown to be 2-3 orders of magnitude faster in comparison to HSPICE. When convolution is avoided for the nonlinear segments, the speed-up is maintained. It is difficult to compare the speed-up to all other nonlinear methods, however, we are able to theoretically estimate the speed-up of this method over the same method requiring convolution.

Let the number of time points needed be N_t, usually $100 \le N_t \le 500$, and let the number of multiplications needed per time point due the number of nonlinear elements and sources be $N_{m \times n}$ which reflects the size of the matrix K_j, and let the order of approximations or the number of poles be N_q, usually $4 \le N_q \le 10$. Then, assuming that we calculated the convolution only at the interface of the sources and the nonlinear elements, and assuming that N_{iter} iterations are needed per nonlinear solution of a time point for both convolution and non-convolution methods, the number of multiplications needed to perform an entire convolution will be generally given by,

$$\sum_{i=1}^{N_t}\sum_{j=1}^{i} N_{iter}N_{m\times n} = \sum_{i=1}^{N_t} iN_{iter}N_{m\times n} = \frac{N_t(N_t+1)N_{iter}N_{m\times n}}{2} \tag{16}$$

Without convolution and using q poles, the number of multiplications is reduced to,

$$\sum_{i=1}^{N_t} N_{iter}N_{m\times n}N_q = N_t N_{iter}N_{m\times n}N_q \tag{17}$$

The ratio of (16) to (17) is,

$$\frac{N_t+1}{N_q}$$

Given typical values of $N_t = 200$ and $N_q = 6$, this is a speed-up of 33. The range of speed-up will generally vary between 25 and 50. This is a conservative estimate since it does not include the speed-up due to avoiding numerical inversion techniques compared to other methods. The following examples demonstrate the relative accuracy of the method.

5. Example

A lossy, coupled transmission line circuit is given in Figure 2. The nonlinear current sources are functions of the output voltage. The source is a 5V pulse of 1ns rise/fall time and 3ns duration. The output voltage is given, first in the lossless, uncoupled case as compared to HSPICE, and then in the lossy, coupled case.

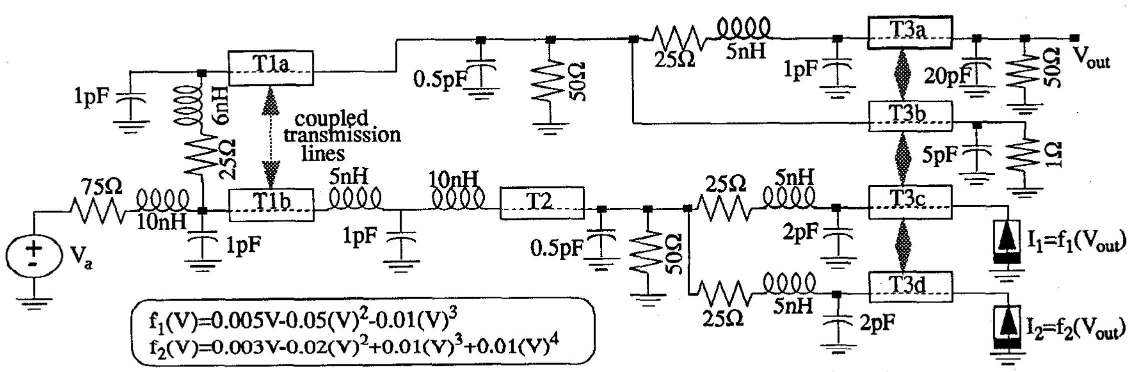


Figure 2: RLCG: nonlinear coupled transmission line network

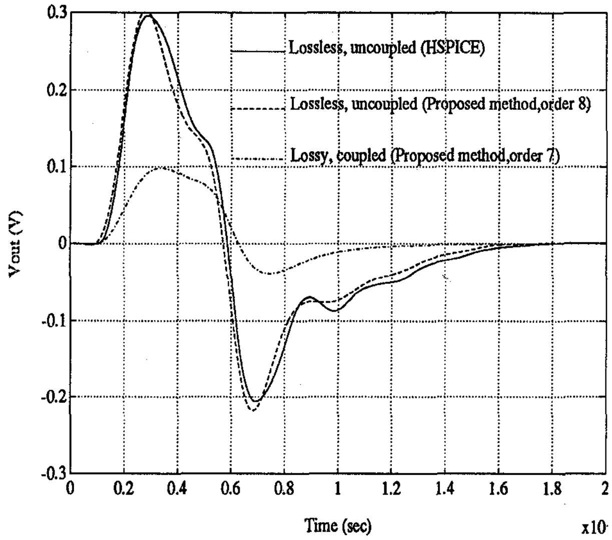


Figure 3: Transient response of Figure 2.

6. Conclusion

The linear waveform estimation of the AWE/GAWE was shown to be 2-3 orders of magnitude faster in linear waveform estimation than a complete analysis by simulators. The method introduced here allows us to use this speed-up for nonlinear waveform estimation and even increase it further by eliminating convolution and/or numerical inversion. Additional refinements of methods used to accurately obtain the dominant linear system modes should further increase the usefulness of the technique.

7.References

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